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HDP/SB/21 based on PTO/SB/21 (08-00)

TRANSMITTAL FORM

(to be used for all correspondence after initial filing)

Application Number	**09/381,839**
Filing Date	September 24, 1999
Inventor(s)	Günter DOEMENS et al.
Group Art Unit	2623
Examiner Name	Colin M. Larose
Attorney Docket Number	32860-000207/US

ENCLOSURES *(check all that apply)*

- ☐ Fee Transmittal Form
  - ☐ Fee Attached
- ☐ Amendment
  - ☐ After Final
  - ☐ Affidavits/declaration(s)
- ☐ Extension of Time Request
- ☐ Express Abandonment Request
- ☐ Information Disclosure Statement
- ☐ Certified Copy of Priority Document(s)
- ☐ Response to Missing Parts/ Incomplete Application
- ☐ Response to Missing Parts under 37 CFR 1.52 or 1.53
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- ☐ Petition
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- ☐ After Allowance Communication to Group
- ☒ LETTER SUBMITTING APPEAL BRIEF AND APPEAL BRIEF (w/clean version of pending claims)
- ☐ Appeal Communication to Group *(Notice of Appeal, Brief, Reply Brief)*
- ☐ Proprietary Information
- ☐ Status Letter
- ☐ Other Enclosure(s) *(please identify below)*:

Remarks

SIGNATURE OF APPLICANT, ATTORNEY, OR AGENT

Firm *or* Individual name	Harness, Dickey & Pierce, P.L.C.	Attorney Name Ray Heflin	Reg. No. 41,060
Signature	Ray Heflin		
Date	December 20, 2005		

PATENT APPLICATION

IN THE U.S. PATENT AND TRADEMARK OFFICE

Appellants:	Günter DOMENS et al.
Application No.:	09/381,839
Art Unit:	2623
Filed:	September 24, 1999
Examiner:	Colin M. Larose
For:	METHOD FOR THREE-DIMENSIONAL IDENTIFICATION OF OBJECTS
Attorney Docket No.:	32860-000207/US

APPELLANTS' AMENDED BRIEF ON APPEAL UNDER 37 CFR §41.37[1]

December 20, 2005

United States Patent and Trademark Office
Customer Service Window, Mail Stop Appeal Brief - Patents
Randolph Building
401 Dulany Street
Alexandria, VA 22314

Sir:

In accordance with the provisions of 37 CFR §41.37, Appellants submit the following:

I. REAL PARTY IN INTEREST:

The real party in interest in this appeal is Siemens Aktiengesellschaft. Assignment of the application was submitted to the U.S. Patent and

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[1] In response to the 12/5/05 Notification of Non-Compliant Appeal Brief, Appellants submit this Amended Brief on Appeal Under 37 CFR §41.37. The only change is the addition of a Related Proceedings Appendix with the indication "none".

Trademark Office on September 24, 1999, and recorded on the same date at Reel 010411, Frame 0964.

II. RELATED APPEALS AND INTERFERENCES:

There are no known appeals or interferences that will affect, be directly affected by, or have a bearing on the Board's decision in this Appeal.

III. STATUS OF CLAIMS:

Claims 4-7 are pending in the application, with claims 4 and 7 being written in independent form. Appellants canceled claims 1-3 via the September 24, 1999 Amendment.

Claims 4-7 remain finally rejected.

Claims 4-7 on appeal are set forth in the attached Claims Appendix.

IV. STATUS OF AMENDMENTS:

No amendments were requested subsequent to the final rejection in the January 12, 2005 Office Action.

V. SUMMARY OF CLAIMED SUBJECT MATTER:

Independent claims 4 and 7 are written in method formats. Both claims are directed to identification of objects (e.g., faces).[2]

A. Claim 4:

With reference to the example, non-limiting embodiment depicted in Fig. 1, the method defined by claim 4 may involve illuminating a digital micro-mirror arrangement 3 via a light source 1.[3] The beam path 8 from the light source 1 may pass through a color filter 2 (which may include red, green and blue areas) to enable color image processing.[4] A color camera 6

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[2] Spec., p. 1, lines 3 and 4.
[3] Spec., p. 5, lines 1-3.
[4] Spec., p. 5, lines 6-10.

and the digital micro-mirror arrangement 3 may be controlled by a control and evaluation unit 11.[5] In this way, the digital micro-mirror arrangement 3 may apply encoded light 10 onto the object surface 7.[6]

The encoded illumination 10 may be projected onto the object surface 7, such that three striped patterns with respectively different colors may be simultaneously present in a video frame.[7] By virtue of the separate and parallel registration of the three different color patterns, the information for calculating three depth planes may be acquired in a video frame.[8] As shown in Fig. 1, the color camera 6 may obtain the video frame (or image of the object surface 7) from a direction that is different from the beam path 8.

The control and evaluation unit 11 may determine a three-dimensional image of a topography of the object surface 7 using triangulation principles.[9] The three-dimensional image and a two-dimensional image of the object may be evaluated.[10]

B. Claim 7:

Claim 7 is substantially similar to claim 4, except that the last clause of claim 7 recites that the three-dimensional image is compared to pre-stored data.[11]

VI. GROUNDS OF REJECTION TO BE REVIEWED ON APPEAL:

Appellants seek the Board's review of the rejection of claims 4-7 under 35 USC §103(a) as being obvious over US 4,511,252 to Di Matteo et al. ("Di

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[5] Spec., p. 5, lines 13 and 14.
[6] Spec., p. 5, lines 14-17.
[7] Spec., p. 5, lines 19-22.
[8] Spec., p. 5, lines 22-24.
[9] Spec., p. 5, lines 29 and 30.
[10] Spec., p. 3, lines 11-13.
[11] See Spec., p. 1, lines 11-15 and p. 6, lines 9-5.

Matteo") in view of US 5,905,545 to Poradish et al. ("Poradish") and US 5,410,609 to Kado et al. ("Kado").

VII. ARGUMENTS:

A. The Obviousness Rejection:

i. Independent Claims 4 and 7:

Each of independent claims 4 and 7 defines a method that involves (among other things) determining a three-dimensional image of a topography of the object surface using at least "*triangulation principles*." As is well known in this art, the term "*triangulation*" refers to the process of finding a distance to a point by calculating the length of one side of a triangle, given measurements of angles and sides of the triangle formed by that point and two other reference points.[12] At least the "*triangulation principles*" feature (as recited in independent claims 4 and 7), in combination with the other features recited in independent claims 4 and 7, is not taught or suggested by the prior art relied upon by the Examiner.

The Examiner relies upon the Di Matteo reference to teach most of the features of the present invention, inclusive of the "*triangulation principles*" feature defined by claims 4 and 7.[13] This rejection position should be reversed for the following reasons.

Di Matteo discloses an arrangement for determining the geometric characteristics of an object. With reference to Fig. 1, the arrangement includes multiple projectors 26, 28, 30 and 32 that project light through variable masks 36 (see Fig. 3) onto a target object 20.[14] At the same time, cameras 40 photograph the surface 22 of the target object 20 within their

[12] The pertinent page from the electronic dictionary *Wikipedia* is provided in the attached Evidence Appendix.
[13] January 12, 2005 Office Action, p. 3 lines 13-19 and p. 5, line 18 – p. 6, line 4.
[14] Di Matteo, col. 4, lines 17-26.

field of view.[15] Turning to Fig. 4, the photographs 44 are scanned by a scanner 46, and the scanned information is inserted into a computer 48.[16] The surface 22 of the target object 20 may be reconstructed from information stored in the computer 48.[17] Di Matteo explains the manner in which the coordinates of points on the surface 22 may be determined by correlating the points to a reference surface (discussed in more detail below). However, the reference does ***not*** provide any disclosure that is pertinent to the claimed "*triangulation principles*" feature. In fact, the reference does not even mention the term triangulation.

The Examiner cites several portions of Di Matteo as allegedly teaching a method that involves determining a three-dimensional image of a topography of the object surface using triangulation principles. However, the heavy reliance upon the cited portions of the reference is misplaced. Each of the cited portions of Di Matteo is discussed separately below.

<u>Column 7, lines 56+:</u> This straightforward portion of Di Matteo indicates that the spatial position of a point on the object surface may be determined using an interpolation technique. The interpolation technique is described with reference to Fig. 8. Here, a photograph of a band of the object surface 22 includes a point P. The image of that band on the photograph appears as an outline 60. A photograph 58 (see Fig. 7) of a reference surface is superimposed onto the photograph containing the outline 60. The spatial position of the point P may then be located by "*interpolation*" with respect to the neighboring points of intersection 58a and 58b.[18] Points 58d and 58c are used for "*interpolating*" to point P from points 58a and 58b, respectively.[19]

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[15] Di Matteo, col. 5, lines 17-20.
[16] Di Matteo, col. 5, lines 53-57.
[17] Di Matteo, col. 6, lines 23-27.
[18] Di Matteo, col. 7, lines 62-65.
[19] Di Matteo, col. 8, lines 1 and 2.

As is well-known in this art, interpolation is a method of constructing new data points (e.g., the point P) from a discreet set of known data points (e.g., 58a, 58b, 58c and 58d).[20] Di Matteo's interpolation technique is simply not pertinent to determining a three-dimensional image of a topography of an object surface using at least "*triangulation principles*," as recited in independent claims 4 and 7.

Column 11, lines 56-68: This portion of the reference merely indicates that the coordinates of a point on a three-dimensional surface may be located by determining the Z coordinate using masks 36 (see Fig. 3), and by determining the X-Y coordinates by scanning a photograph 44 (see Fig. 2) of the surface and superimposing a reticle 82 (see Fig. 12) onto the photograph 44.

This disclosure is not at all pertinent to "*triangulation principles*." The Examiner's assertions to the contrary are simply incorrect.

Column 14, lines 57-68 and column 15, lines 1-12: This portion of Di Matteo explains an embodiment (depicted in Fig. 18) in which the location of a point on a surface may be determined from "*computations involving the intersection of a line and a plane.*"[21] Here, if a point on a surface is photographed and its image appears on the film 106 (at point P_3), the surface point of interest must lie somewhere along the line 110 (extending between the lens node 108 and the point P_2 on the reference surface 52).[22] Every known point on the reference surface 52 will define a known line between itself and the lens node 108 (e.g., line 110) and will further define a point on the film (e.g., P_3).[23] Thus, a picture of the reference surface 52 may be used as an overlay to the pictures of an unknown surface to "*transform positions*" on the film to lines through the lens node that are geometrically

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[20] The pertinent page from the electronic dictionary *Wikipedia* is provided in the attached Evidence Appendix.
[21] Di Matteo, col. 13, lines 41-44.
[22] Di Matteo, col. 14, lines 50-53.
[23] Di Matteo, col. 14, lines 53-57.

exact.[24] In practice, the spatial coordinates may be assigned to a point P of interest, since that point may be solved for as the intersection of a plane and the line obtained from the position of the image and that point on the film.[25]

Appellants contend that such teachings are simply not pertinent to the "*triangulation principles*" feature recited in independent claims 4 and 7. The Examiner assertions to the contrary appear to based upon a speculation and/or a misunderstanding of the reference.

Turning to the next point, the Examiner makes assertions concerning Fig. 8 of Di Matteo in an attempt to bolster the rejection position. Some of the assertions warrant further comment as follows. First, the Examiner points out that Di Matteo's entire disclosure is devoted to ascertaining the 3-D geometrical dimensions of objects and relies on triangulation principles such as shown in Fig. 8.[26] However, as discussed in detail above, Fig. 8 of Di Matteo illustrates how the spatial position of a point on the object surface may be determined using an interpolation technique (which is practically and conceptually different than triangulation principles).[27] The allegation that Di Matteo's "*entire disclosure*" is devoted to ascertaining geometrical dimensions does not somehow change or detract from the straightforward and express disclosure directed to Fig. 8 of the reference.

Second, Appellants agree with the Examiner to the extent that triangulation techniques may involve (among other things) using trigonometric principles to determine the coordinates of a point in space, where the point may be the vertex of a triangle having two other vertices with known coordinates.[28] However, the Examiner's contention that Fig. 8 of Di Matteo fairly teaches a triangulation technique is without merit for the

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[24] Di Matteo, col. 14, lines 57-60.
[25] Di Matteo, col. 15, lines 1-12.
[26] June 14, 2005 Advisory Action, Item 11.
[27] Di Matteo, col. 7, lines 56+.
[28] June 14, 2005 Advisory Action, Item 11.

reasons discussed above (i.e., Fig. 8 is directed to an interpolation technique, but not a triangulation technique).

Third, in an effort to support the alleged triangulation theory, the Examiner contends that in Fig. 8 of Di Matteo, the location of point P is ascertained by forming a triangle composed of two other known points.[29] This simply not understood because Fig. 8 does not illustrate a triangle. Instead, the drawing contains an oval shape (formed by intersections 58), an arcuate outline 60, and two straight lines that intersect at point P.

ii. Conclusion:

As demonstrated above, the Di Matteo does not teach or suggest the features upon which the Examiner relies to reject the claims. Further, the secondary references to Poradish and Kado are not believed to make up for the deficiencies of Di Matteo noted above. Accordingly, even if combined in the manner suggested by the Examiner, the prior art would still not meet each and every feature of the invention defined by independent claims 4 and 7.

The Commissioner is authorized in this, concurrent, and future replies, to charge payment or credit any overpayment to Deposit Account No. 08-0750 for any additional fees required under 37 C.F.R. § 1.16 or under 37 C.F.R. § 1.17; particularly, extension of time fees.

Respectfully submitted,

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DJD/HRH

[29] June 14, 2005 Advisory Action, Item 11.

CLAIMS APPENDIX

Claims 4-7 on Appeal:

4. A method for identification of an object having an object surface, said method comprising:

illuminating a digital micro-mirror arrangement via a light source;

successively projecting a number of encoded illumination patterns by driving said digital micro-mirror arrangement to sequentially illuminate said object surface, with the digital micro- mirror arrangement being sequentially illuminated with at least three colors in a beam path through a variable color filter onto said object surface for identification of at least three depth planes of said object in a single image;

registering said image of said object with a color camera from a direction different from said beam path;

determining a three-dimensional image of a topography of said object surface from said registration in a control and evaluation unit, the determining including the use of at least triangulation principles; and

evaluating the three-dimensional image and a two-dimensional image of said object.

5. The method according to claim 4, wherein said encoded illumination patterns comprise a stripe pattern having successively varied periodicity.

6. The method according to claim 4, wherein said method is used for face identification.

7. A method for identification of an object having an object surface, said method comprising:

illuminating a digital micro-mirror arrangement via a light source;

successively projecting a number of encoded illumination patterns by driving said digital micro-mirror arrangement to sequentially illuminate said object surface, with the digital micro- mirror arrangement being sequentially illuminated with at least three colors in a beam path through a variable color filter onto said object surface for identification of at least three depth planes of said object in a single image;

registering said image of said object with a color camera from a direction different from said beam path;

determining a three-dimensional image of a topography of said object surface from said registration in a control and evaluation unit, the determining including the use of at least triangulation principles; and

comparing the three-dimensional image to pre-stored data.

EVIDENCE APPENDIX

1. One page from the electronic dictionary *Wikipedia* for the term "triangulation."
2. Five pages from the electronic dictionary *Wikipedia* for the term "interpolation."

RELATED PROCEEDINGS APPENDIX

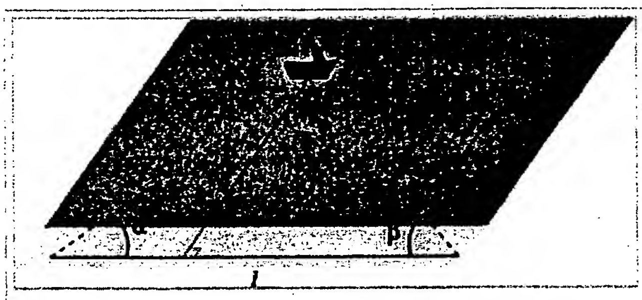
None.

Triangulation

From Wikipedia, the free encyclopedia.

This article is about measurement by the use of triangles: for other usages of the term "triangulation", see triangulation (disambiguation).

In trigonometry and elementary geometry, **triangulation** is the process of finding a distance to a point by calculating the length of one side of a triangle, given measurements of angles and sides of the triangle formed by that point and two other reference points.



Triangulation can be used to find the distance from the shore to the ship. The observer at α measures the angle between the shore and the ship, and the observer at β does likewise. If the length l is known, then the law of sines can be applied to find the distance d.

Some identities often used (valid only in flat or euclidean geometry):

- The sum of the angles of a triangle is π rad or 180 degrees.
- The law of sines
- The law of cosines
- The Pythagorean theorem

Triangulation is used for many purposes, including surveying, navigation, astrometry, binocular vision and gun direction of weapons.

Many of these surveying problems involve the solution of large meshes of triangles, with hundreds or even thousands of observations. Complex triangulation problems involving real-world observations with errors require the solution of large systems of simultaneous equations to generate solutions.

Famous uses of triangulation have included the retriangulation of Great Britain.

See also

- Parallax
- Trilateration, wherein a point is calculated given its distances from other known points

Retrieved from "http://en.wikipedia.org/wiki/Triangulation"

Categories: Elementary geometry | Euclidean geometry | Surveying | Angle

Interpolation

From Wikipedia, the free encyclopedia.

This article is about interpolation in mathematics. See also interpolation (music) and interpolation (manuscripts).

In the mathematical subfield of numerical analysis, **interpolation** is a method of constructing new data points from a discrete set of known data points.

In engineering and science one often has a number of data points, as obtained by sampling or some experiment, and tries to construct a function which closely fits those data points. This is called curve fitting. Interpolation is a specific case of curve fitting, in which the function must go exactly through the data points.

A different problem which is closely related to interpolation is the approximation of a complicated function by a simple function. Suppose we know the function but it is too complex to evaluate efficiently. Then we could pick a few known data points from the complicated function and try to interpolate those data points to construct a simpler function. Of course when using the simple function to calculate new data points we usually do not receive the same result as when using the original function, but depending on the problem domain and the interpolation method used the gain in simplicity might offset the error.

It should be mentioned that there is also another very different kind of interpolation in mathematics, namely the ``interpolation of operators". The classical results about interpolation of operators are the Riesz-Thorin theorem and the Marcinkiewicz theorem and there are many other subsequent results.

Contents

Definition

Given a sequence of n *distinct* numbers x_k called **nodes** and for each x_k a second number y_k, we are looking for a function f so that

$$f(x_k) = y_k \ , k = 1, \ldots, n$$

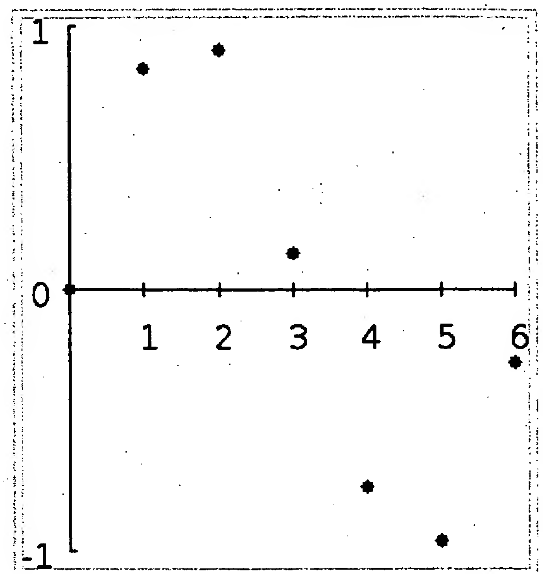
A pair x_k, y_k is called a **data point** and f is called the **interpolant** for the data points.

When the y_k are given by a known function we sometimes write f_k.

Example

For example, suppose we have a table like this, which gives some values of an unknown function *f*.

x	*f*(*x*)
0	0
1	0.8415
2	0.9093
3	0.1411
4	−0.7568
5	−0.9589
6	−0.2794



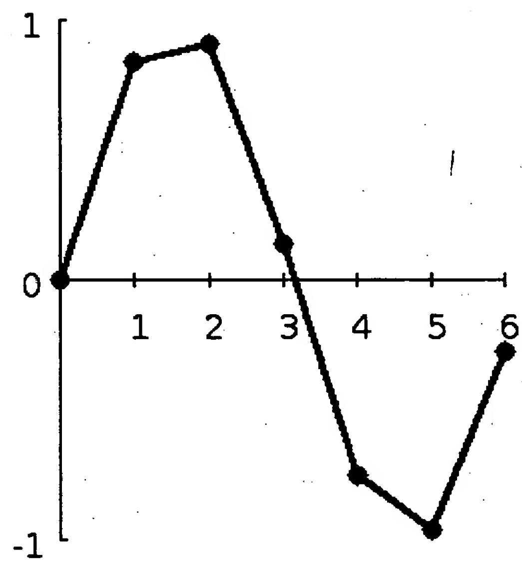
Plot of the data points as given in the table.

What value does the function have at, say, $x = 2.5$? Interpolation answers questions like this.

There are many different interpolation methods, some of which are described below. Some of the things to take into account when choosing an appropriate algorithm are: How accurate is the method? How expensive is it? How smooth is the interpolant? How many data points are needed?

Linear interpolation

Main article: Linear interpolation



One of the simplest methods is linear interpolation (sometimes known as lerp). Consider the above example of determining $f(2.5)$. Since 2.5 is midway between 2 and 3, it is reasonable to take $f(2.5)$ midway between $f(2) = 0.9093$ and $f(3) = 0.1411$, which yields 0.5252.

Generally, linear interpolation takes two data points, say (x_a, y_a) and (x_b, y_b), and the interpolant is given by

$$f(x) = \frac{x - x_b}{x_a - x_b} y_a - \frac{x - x_a}{x_a - x_b} y_b$$

This formula can be interpreted as a weighted mean.

Linear interpolation is quick and easy, but it is not very precise. Another disadvantage is that the interpolant is not differentiable at the point x_k.

The following error estimate shows that linear interpolation is not very precise. Denote the function which we want to interpolate by *g*, and suppose that *x* lies between x_a and x_b and that *g* is twice continuously differentiable. Then the linear interpolation error is

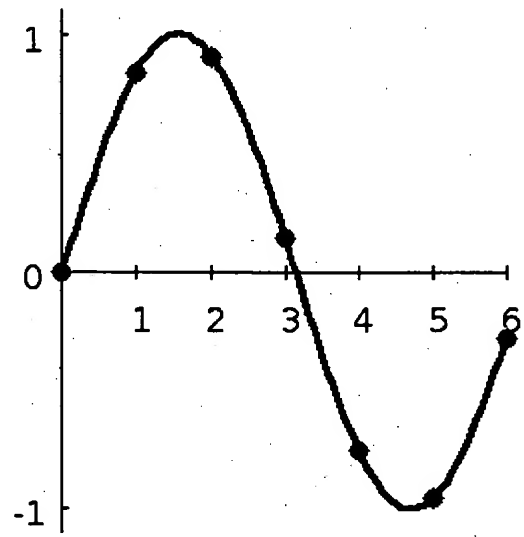
$$|f(x) - g(x)| \le C(x_b - x_a)^2 \quad \text{where} \quad C = \frac{1}{8} \max_{y \in [x_a, x_b]} g''(y).$$

In words, the error is proportional to the square of the distance between the data points. The error of some other methods, including polynomial interpolation and spline interpolation (described below), is proportional to higher powers of the distance between the data points. These methods also produce smoother interpolants.

Polynomial interpolation

Main article: Polynomial interpolation

Polynomial interpolation is a generalization of linear interpolation. Note that the linear interpolant is a linear function. We now replace this interpolant by a polynomial of higher degree.



Consider again the problem given above. The following sixth degree polynomial goes through all the seven points:

$$f(x) = -0.0001521x^6 - 0.003130x^5 + 0.07321x^4 - 0.3577x^3 + 0.2255x^2 + 0.9038x.$$

Substituting $x = 2.5$, we find that $f(2.5) = 0.5965$.

Generally, if we have n data points, there is exactly one polynomial of degree $n-1$ going through all the data points. The interpolation error is proportional to the distance between the data points to the power n. Furthermore, the interpolant is a polynomial and thus infinitely differentiable. So, we see that polynomial interpolation solves all the problems of linear interpolation.

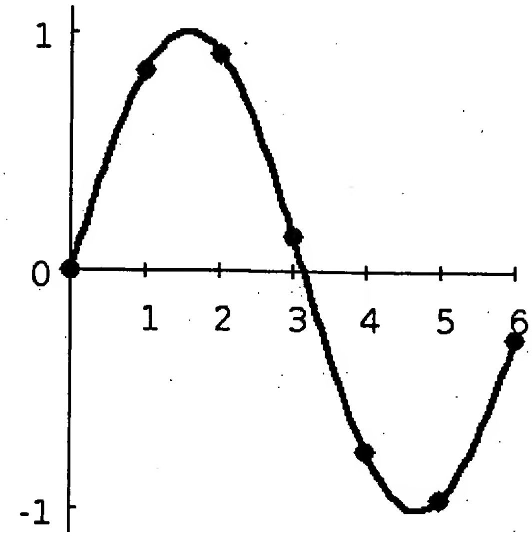
However, polynomial interpolation also has some disadvantages. Calculating the interpolating polynomial is very 'expensive' (in relative terms of computer calculation time). Furthermore, polynomial interpolation may not be so exact after all, especially at the end points (see Runge's phenomenon). These disadvantages can be avoided by using spline interpolation.

Spline interpolation

Main article: spline interpolation

Remember that linear interpolation uses a linear function for each of intervals $[x_k, x_{k+1}]$. Spline interpolation uses low-degree polynomials in each of the intervals, and chooses the polynomial pieces such that they fit smoothly together. The resulting function is called a spline.

For instance, the natural cubic spline is piecewise cubic and twice continuously differentiable. Furthermore, its second derivative is zero at the end points. The natural cubic spline interpolating the points in the table above is given by



$$f(x) = \begin{cases} -0.1522x^3 + 0.9937x, & \text{if } x \in [0,1], \\ -0.01258x^3 - 0.4189x^2 + 1.4126x - 0.1396, & \text{if } x \in [1,2], \\ 0.1403x^3 - 1.3359x^2 + 3.2467x - 1.3623, & \text{if } x \in [2,3], \\ 0.1579x^3 - 1.4945x^2 + 3.7225x - 1.8381, & \text{if } x \in [3,4], \\ 0.05375x^3 - 0.2450x^2 - 1.2756x + 4.8259, & \text{if } x \in [4,5], \\ -0.1871x^3 + 3.3673x^2 - 19.3370x + 34.9282, & \text{if } x \in [5,6]. \end{cases}$$

Like polynomial interpolation, spline interpolation incurs a smaller error than linear interpolation and the interpolant is smoother. However, the interpolant is easier to evaluate than the high-degree polynomials used in polynomial interpolation. It also does not suffer from Runge's phenomenon.

Other forms of interpolation

Other forms of interpolation can be constructed by picking a different class of interpolants. For instance, rational interpolation is **interpolation** by rational functions, and trigonometric interpolation is interpolation by trigonometric polynomials. The discrete Fourier transform is a special case of trigonometric interpolation. Another possibility is to use wavelets.

The Nyquist-Shannon interpolation formula can be used if the number of data points is infinite.

Multivariate interpolation is the interpolation of functions of more than one variable. Methods include bilinear interpolation and bicubic interpolation in two dimensions, and trilinear interpolation in three dimensions.

Sometimes, we know not only the value of the function that we want to interpolate, at some points, but also its derivative. This leads to Hermite interpolation problems.

Related concepts

The term *extrapolation* is used if we want to find the value of f at a point x which is outside of the points x_k at which f is given.

In curve fitting problems, the constraint that the interpolant has to go exactly through the data points is relaxed.

We only require that it approaches the data points as closely as possible. This requires parameterizing the potential interpolants and having some way of measuring the error. In the simplest case this leads to least squares approximation.

Approximation theory studies how to find the best approximation to a given function by another function from some predetermined class, and how good this approximation is. This clearly yields a bound on how well the interpolant can approximate the unknown function.

References

- David Kidner, Mark Dorey and Derek Smith (1999). *What's the point? Interpolation and extrapolation with a regular grid DEM (http://www.geovista.psu.edu/sites/geocomp99/Gc99/082/gc_082.htm).* IV International Conference on GeoComputation, Fredericksburg, VA, USA.
- David Kincaid and Ward Cheney (2002). *Numerical Analysis* (3rd ed.), Chapter 6. Brooks/Cole. ISBN 0-534-38905-8.
- Michelle Schatzman (2002). *Numerical Analysis: A Mathematical Introduction,* Chapters 4 and 6. Clarendon Press, Oxford. ISBN 0-19-850279-6.

External links

- Digital Image Interpolation (http://www.cambridgeincolour.com/tutorials/image-interpolation.htm) : Fundamental understanding for digital images